# Math 115 - Quiz 2

February 8, 2013

**Problem 1** (4 points): Let

$$f(x) = \begin{cases} (x^2 - 4)\,/\,(x+2) & \text{if } x \neq -2 \\ k & \text{if } x = -2 \end{cases}$$

For what value of $k$ will $f$ be continuous on $(-\infty, \infty)$.

**Problem 2** (4 points): Under a set of controlled laaboratory conditions, the size of the population of a certain bacteria culture at time $t$, in minutes, is described by the function

$$P = f(t) = 3t^2 + 2t + 1$$

Find the rate of population growth at $t = 10$ minutes.

1) $\lim_{x \to -2} \frac{(x^2-4)}{(x+2)} = \lim_{x \to -2} \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = \lim_{x \to -2} (x-2) = -4$

$k = -4$

2) $f'(t) = 6t + 2$

$f'(10) = 6(10) + 2 = 62$

**Problem 3** (2 points): Prove

$$\frac{d}{dx}\left[f\left(x\right)+g\left(x\right)\right]=\frac{d}{dx}\left[f\left(x\right)\right]+\frac{d}{dx}\left[g\left(x\right)\right]$$

*Hint*: Set $S(x) = f(x) + g(x)$. Take the limit definition of the derivative of $S(x)$ and then swap $f(x) + g(x) = S(x)$ back in for $S(X)$. From there, use the properties of limits to get what you're looking for.

**Bonus** (2 points): Use the Intermediate Value Theorem to show that there exists a number $c$ in the given interval such that $f(x) = M$. Then find its value.

$$f(x) = x^2 - x + 1 \text{ on } [-1, 4]; \; M = 7$$

3) let $S(x) = f(x) + g(x)$

$$\frac{d}{dx}[S(x)] = \lim_{h\to 0} \frac{S(x+h) - S(x)}{h}$$

$$= \lim_{h\to 0} \frac{(f(x+h) + g(x+h)) - (f(x) - g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right]$$

$$= \lim_{h\to 0} \left[\frac{f(x+h) - f(x)}{h}\right] + \lim_{h\to 0} \left[\frac{g(x+h) - g(x)}{h}\right)$$

$$= \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

B) $f(-1) = (-1)^2 - (-1) + 1 = 3$

$f(4) = (4)^2 - (4) + 1 = 13$

$3 < 7 < 13$

$3 < M < 13$

so there exists a c so that

$f(c) = M$

# Quiz 4 (10 pts)

find the derivative of

$$f(x) = \frac{x^4 - 3x^2 + 2}{x^2 - 2}$$

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$$f'(x) = \frac{(4x^3 - 6x)(x^2-2) - (x^4 - 3x^2 + 2)(2x)}{(x^2-2)^2}$$

$$g(x) = \frac{1}{\sqrt{3x^2 + 2x + 2}} = (3x^2 + 2x + 2)^{-1/2}$$

$$f'(x) = -1/2\,(3x^2 + 2x + 2)^{-3/2}(6x + 2)$$

$$h(x) = \sqrt{3x^2 - 2x + 1} = (3x^2 - 2x + 1)^{1/2}$$

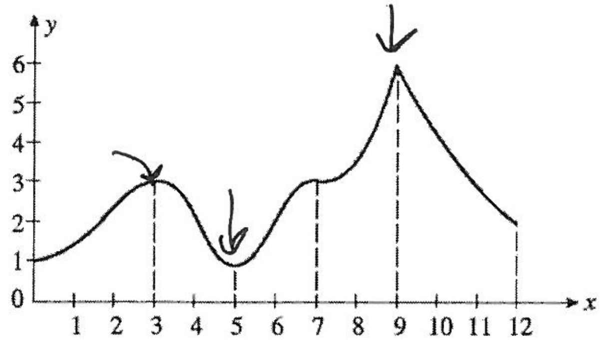
$$f'(x) = 1/2\,(3x^2 - 2x + 1)^{-1/2}(6x - 2)$$

$$=$$

# Math 115 - Quiz 5

March 27, 2013

**Problem 1**
**(a)** On the following graph, which of the following points are relative extrema?
(1 point per correct, -0.5 per wrong, 3 points total)




| | | | |
|---|---|---|---|
| (I) (1,1) | (II) (2,2) | (III) (3,3) | (IV) (4,2) |
| (V) (5,1) | (VI) (6,2) | (VII) (7,3) | (VIII) (8,4) |
| (IX) (9,6) | (X) (10,4) | (XI) (11,3) | (XII) (12,2) |

**(b)** Draw the sign diagram for $f'$.
(2 points)

++++++++ ---- +++++++++ ----

3 5 9

**Problem 2** Determine the regions where the graph of $f(x) = x^3 - x^2 - 5x + 8$ is concave upward, concave downward, increasing, and decreasing.
(5 points)

$f'(x) = 3x^2 - 2x - 5$

$f''(x) = 6x - 2$

$0 = 6x - 2$

$x = 1/3$

$0 = 3x^2 - 2x - 5$

$0 = 3x^2 + 3x - 5x - 5$

$0 = 3x(x+1) - 5(x+1)$

$0 = (3x - 5)(x+1)$

$x = -1, 5/3$

| | TV | $f'(x)$ | |
|---|---|---|---|
| $(-\infty, -1)$ | -2 | + | inc |
| $(-1, 5/3)$ | 0 | - | dec |
| $(5/3, \infty)$ | 2 | + | inc |

| | TV | $f''(x)$ | |
|---|---|---|---|
| $(-\infty, 1/3)$ | 0 | - | down |
| $(1/3, \infty)$ | 1 | + | up |

# Math 115 - Quiz 6

April 4, 2013

**Problem 1** Solve for $x$.

(a) $3^{x-x^2} = 1/9^x$ (b) $3^{2x} - 12 \cdot 3^x + 27 = 0$

**Problem 2** Simplify.

(a) $\frac{4b^{-4}}{12b^{-6}}$ (b) $\left(x^{-b/a}\right)^{-a/b}$

1a) $3^{x-x^2} = (9^{-1})^x$

$3^{x-x^2} = (3^{-2})^x$

$3^{x-x^2} = 3^{-2x}$

$x - x^2 = -2x$

$3x - x^2 = 0$

$x(3-x) = 0$

$x=0$ or $x=3$

1b) let $U = 3^x$

$3^{2x} - 12 \cdot 3^x + 27 = 0$

$U^2 - 12U + 27 = 0$

$(U-9)(U-3) = 0$

$U=9$ $U=3$

$3^x = 9$ $3^x = 3$

$x=2$ $x=1$

2a) $\frac{4}{12} b^{-4-(-6)}$

$\frac{1}{3} b^2$

2b) $\left(x^{-b/a}\right)^{-a/b}$

$= x^{(-b/a)(-a/b)}$

$= x^{\frac{ba}{ab}}$

$= x^1$

$= x$

# Math 115 - Quiz 7

April 12, 2013

**Problem** (2 points each) Find the derivative of the following functions:

| | | | |
|---|---|---|---|
| (a) | $f(x) = x^3 e^x$ | (b) | $g(x) = 3e^x$ |
| (c) | $h(x) = \ln\left(x^3+1\right)$ | (d) | $k(x) = x^2 \ln(x)$ |
| (e) | $l(x) = \ln[(x+1)(x^3+2)]$ | | |

**Bonus problem** (5 points) Use logarithmic differentiation to find the derivative of the following:

$$y = (x-1)^2(x+1)^3(x+3)^4$$

(a) $f'(x) = (3x^2)e^x + (e^x)(x^3)$

product rule

(b) $g'(x) = \frac{\partial}{\partial x}[3e^x] = 3\frac{\partial}{\partial x}[e^x] = 3e^x$

(c) $h'(x) = \dfrac{\frac{d}{dx}[x^3+1]}{x^3+1} = \dfrac{3x^2}{x^3+1}$

(d) $(2x)\ln x + \left(\frac{1}{x}\right)(x^2)$

product rule $= 2x\ln x + x = x(2\ln x + 1)$

(e) $\ln[(x+1)(x^3+2)] = \ln(x+1) + \ln(x^3+1)$

$l'(x) = \dfrac{1}{x+1} + \dfrac{3x^2}{x^3+1}$

$\ln y = \ln\left[(x-1)^2(x+1)^3(x+3)^4\right]$

$= \ln(x-1)^2 + \ln(x+1)^3 + \ln(x+3)^4$

$= 2\ln(x-1) + 3\ln(x+1) + 4(\ln(x+3))$

$\frac{\partial}{\partial x}[\ln y] = 2\frac{1}{x-1} + 3\frac{1}{x+1} + 4\frac{1}{x+3}$

$\dfrac{y'}{y} = \dfrac{2}{x-1} + \dfrac{3}{x+1} + \dfrac{4}{x+3}$

$y' = y\left(\dfrac{2}{x-1} + \dfrac{3}{x+1} + \dfrac{4}{x+3}\right)$

$= (x-1)^2(x+1)^3(x+3)^4\left(\dfrac{2}{x-1} + \dfrac{3}{x+1} + \dfrac{4}{x+3}\right)$

# Math 115 - Quiz 8

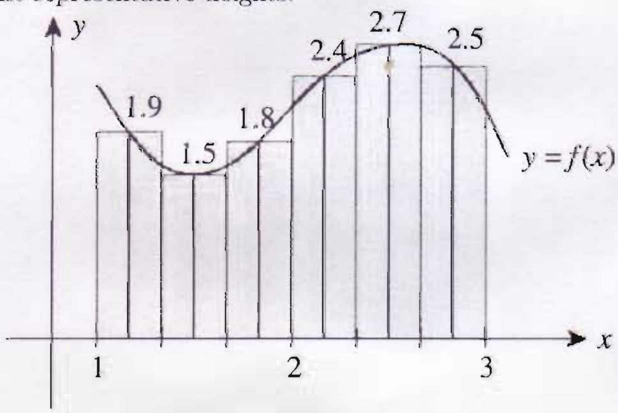
April 26, 2013

**Problem 1** (5 points) Phosphorus 32 (P-32) has a half-life of 14.2 days. If 100g of this substance are present initially, find the amount present after $t$ days. What will be left after 7.1 days?

**Problem 2** (5 points) Find the indefinite integral:

$$\int x^2 + x - x^{-3}\,dx$$

**Bonus problem** (5 points) Find an approximation of the area of the region under the graph of $f(x)$ by computing the Riemann sum. Use the given midpoints as the representative heights:




1) $Q(t) = Q_0 e^{kt}$

$50 = 100e^{k(14.2)}$

$1/2 = e^{k(14.2)}$

$\ln(1/2) = k\,14.2$

$k = \frac{\ln(1/2)}{14.2}$

$\approx -0.0488$

$Q(7.1) \approx 100e^{-0.0488(7.1)}$

$\approx 70.71$

2) $\int x^2 + x - x^{-3}\,dx$

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{2}x^{-2}$

bonus) rectangles $(n) = 6$

$\Delta x = \frac{3-1}{6} = 1/3$

area =

$= 1/3(1.9 + 1.5 + 1.8 + 2.4 + 2.7 + 2.5)$

$= 4.2\overline{6}$

# Math 115 - Quiz 9

May 3, 2013

**Problem 1** (5 points) Find the following indefinite integrals:

$(a)$

$$\int \left(2 + x + 2x^2 + e^x\right)\ dx$$

$(b)$

$$\int \frac{1}{x} + \frac{1}{x^2}\ dx$$

**Problem 2** (5 points) Find the value of the following definite integrals:

$(a)$

$$\int_1^3 \frac{2}{x}\ dx$$

$(b)$

$$\int_1^8 4x^{1/2} + 3\ dx$$

**Bonus problem** (5 points) Evaluate using $u$-substitution:

$$\int_{-1}^{1} x^2 \left(x^3 + 1\right)^4\ dx$$

1a) $\int 2 + x + 2x^2 + e^x \, dx$

$= 2x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + e^x + C$

1b) $\int \frac{1}{x} + \frac{1}{x^2}\, dx$

$= \ln|x| + \frac{-1}{x} + C$

2a) $\int_1^3 \frac{2}{x}\, dx = 2\int_1^3 \frac{1}{x}\, dx$

$= 2 \ln|x| \Big|_1^3 = 2(\ln 3 - \ln 1)$

$= 2\ln 3 \approx 2.197$

2b) $\int_1^8 4x^{1/2} + 3\, dx = 4\frac{2}{3}x^{3/2} + 3x \Big|_1^8$

$= \left(\frac{8}{3}(8)^{3/2} + 3(8)\right) - \left(\frac{8}{3}(1)^{3/2} + 3(1)\right)$

$\approx 78.67$

Bonus

let $u = x^3 + 1$

$\frac{du}{dx} = 3x^2 \quad du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$x = -1 \to u = (-1)^3 + 1 = 0$

$x = 1 \to u = (1)^3 + 1 = 2$

$\int_{u=0}^{u=2} u^4 \frac{1}{3}\, du = \frac{1}{3}\int_0^2 u^4\, du$

$= \frac{1}{3}\frac{1}{5} u^5 \Big|_0^2 = \frac{1}{15}(2^5 - 0^5)$

$= 2.13\overline{3}$